Eur Phys J A (2005) **24,** s1, 3–8
DOI: 10.1140/epjad/s2005-05-001-3

EPJ A direct
electronic only

Relativistic mean field approximation to baryons

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Received: 22 October 2004 / Published Online: 8 February 2005

Abstract. We stress the importance of the spontaneous chiral symmetry breaking for understanding the low-energy structure of baryons. The Mean Field Approximation to baryons is formulated, which solves several outstanding paradoxes of the naive quark models, and which allows to compute parton distributions at low virtuality in a consistent way. We explain why this approach to baryons leads to the prediction of relatively light exotic pentaquark baryons, in contrast to the constituent models which do not take seriously the importance of chiral symmetry breaking. We briefly discuss why, to our mind, it is easier to produce exotic pentaquarks at low than at high energies.

PACS. 12.38.-t – 12.39.-x – 12.39.Dc – 14.20-c

1 Spontaneous chiral symmetry breaking (SCSB)

The crucial aspect of QCD is the spontaneous breaking of the chiral symmetry: as the result the nearly massless "bare" or "current" u, d, s quarks obtain a dynamical, momentum-dependent mass $M(p)$ with $M(0) \approx 350\,\text{MeV}$ for the u, d quarks and $\approx 470\,\text{MeV}$ for the s quark. The microscopic mechanism of how light quarks become heavy, including the above numbers, can be understood as due to instantons [1] – large fluctuations of the gluon field in the vacuum, needed to make the $\eta'(958)$ meson heavy [2]. Instantons are specific fluctuations of the gluon field that are capable of capturing light quarks, see [3] for a recent review. Quantum-mechanically, quarks can hop from one instanton to another each time flipping the helicity. When it happens many times quarks obtain the dynamical mass $M(p)$. This mass goes to zero at large momenta since quarks with very high virtuality are not affected by any background, even if it is a strong gluon field as in the case of instantons. Instantons may not be the only and the whole truth but the mechanism of the SCSB as due to the delocalization of the zero quark modes in the vacuum [1] is probably here to stay.

The appearance of the dynamical mass $M(p)$ is instrumental in understanding the world of hadrons made of the u, d, s quarks. Indeed, the normal lowest lying vector mesons have approximately twice this mass while the ground-state baryons have the mass of approximately thrice M. It does not mean that they are weakly bound: as usual in quantum mechanics, the gain in the potential energy of a bound system is to a big extent compensated by the loss in the kinetic energy, as a consequence of the uncertainty principle. Therefore, one should expect the size of light hadrons to be of the scale of $1/M \approx 0.7\,\text{fm}$, which indeed they are. At the same time the size of the constituent quarks is roughly given by the slope of $M(p)$, corresponding to about $\frac{1}{3}\,\text{fm}$. Therefore, constituent quarks in hadrons are generally well separated, which is a highly non-trivial fact. It explains why the constituent quark idea has been a useful guideline for 40 years.

2 Mesons

In the language of the Dirac spectrum for quarks, vector, axial and tensor mesons are the particle-hole excitations of the vacuum, see Fig. 1. In the Dirac theory, a hole in the negative-energy continuum is the absence of a quark

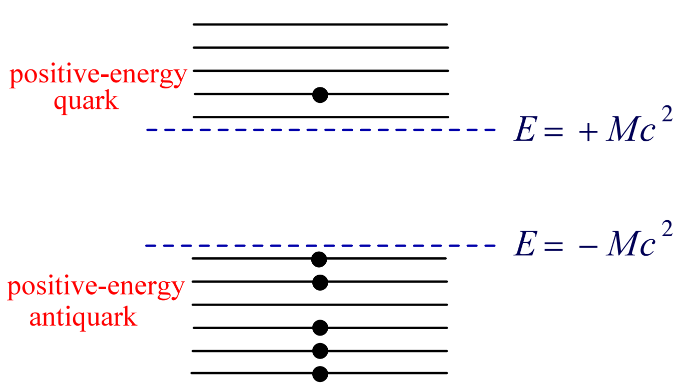

Fig. 1. Vector mesons are particle-hole excitations of the vacuum. They are made of a quark with positive energy and an antiquark with positive energy, hence their mass is roughly $2M$

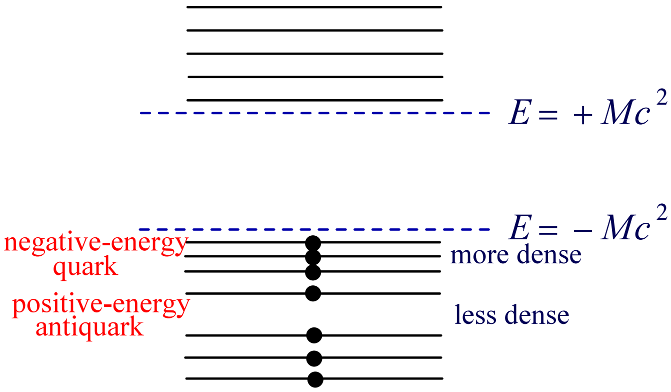

Fig. 2. Pseudoscalar mesons are *not* particle-hole excitations but a collective re-arrangement of the vacuum. They are made of an antiquark with positive energy and a quark with *negative* energy, hence their mass is roughly zero

with negative energy, or the presence of an antiquark with positive energy. To create such an excitation, one has to knock out a quark from the sea and place it in the upper continuum: that costs minimum $2M$ in a non-interacting case, and gives the scale of the vector (as well as axial and tensor) meson masses in the interacting case as well.

For pions, this arithmetic fails: their mass is zero by virtue of the Goldstone theorem. One can say that in pions twice the constituent quark mass is completely eaten up by a strong interaction (which is correct) but there is a more neat way to understand it.

Pseudoscalar mesons are totally different in nature from, say, the vector mesons. They are Goldstone bosons associated with symmetry breaking. A chiral rotation costs zero energy: it is the same vacuum state. Pseudoscalar mesons are described by the same filled Dirac sea with negative energies as the vacuum state. They are not particle-hole excitations. If the Goldstone boson carries some energy, it corresponds to a slightly distorted spectral density of the Dirac sea (Fig. 2). The region of the Dirac sea where the level density is lower than in the vacuum, is a hole and corresponds to an antiquark with positive energy. The region with higher density than in the vacuum corresponds to an extra quark with a negative energy, since there are now "more quarks" in the negative-energy Dirac sea. Therefore, the pseudoscalar mesons are "made of" a *positive-energy antiquark* and a **negative**-*energy quark*. The mass is hence $(M - M) = 0$. This explains why their mass is zero in the chiral limit, or close to zero if one recalls the small u, d, s bare masses which break explicitly chiral symmetry from the start.

3 Baryons

Without spontaneous chiral symmetry breaking, the nucleon would be either nearly massless or degenerate with its chiral partner, $N(1535, \frac{1}{2}^-)$. Both alternatives are many hundreds of MeV away from reality, which serves as one of the most spectacular experimental indications that chiral symmetry is spontaneously broken. It also serves as a warning that if we disregard the effects of the SCSB we shall get nowhere in understanding baryons.

Reducing the effects of the SCSB to ascribing quarks a dynamical mass of about 350 MeV and verbally adding that pions are light, is, however, insufficient. In fact it is inconsistent to stop here: one cannot say that quarks get a constituent mass but throw out their strong interaction with the pion field. Constituent quarks necessarily have to interact with pions, as a consequence of chiral symmetry, and actually very strongly. I have had an opportunity to talk about it recently [4] and shall not repeat it here.

Inside baryons, quarks experience various kinds of interactions: colour Coulomb, colour spin-spin (or hyperfine) and the interaction with the chiral field mentioned above. It is important to know which interaction is stronger and which one is weaker and can be disregarded in the first approximation. A simple estimate using the running α_s at typical interquark separations shows that the chiral force is, numerically, the strongest one. There is also a theoretical argument in its favor. Taking, theoretically, the large-N_c (the number of colours) limit has been always considered as a helpful guideline in hadron physics. It is supposed that if some observable is stable in this academic limit, then in the real world with $N_c = 3$ it does not differ strongly from its limiting value at $N_c \to \infty$. There are many calculations, both analytical and on the lattice, supporting this view. Therefore, if a quantity is stable in the large-N_c limit, one has to be able to get it from physics that survives at large N_c. At arbitrary N_c, baryons are made of N_c constituent quarks sharing the same s-wave orbital but antisymmetrized in colour. Baryons' masses grow linearly with N_c but their sizes are stable in N_c [6]. It means that one has to be able to obtain the quark wave function in the large-N_c limit, and that presumably it will not differ more than by a few percent from the true wave function at $N_c = 3$.

When the number of participants is large, one usually applies the mean field approximation to bound states, the examples being the Thomas–Fermi approximation to atoms and the shell model for nuclei. In these two examples the large number of participants are distributed in many orbitals or shells, whereas in the nucleon all participants are in one orbital. This difference is in favor of the nucleon as one expects smaller corrections from the fluctuations about the mean field in this case. [Indeed, corrections to the Thomas–Fermi approximation are known to die out as $1/\sqrt{Z}$ whereas for nucleons they die out faster as $1/N_c$.]

If the mean field is the colour one, it has to point out in some direction in the colour space. Hence the gluon field cannot serve as the mean field without breaking colour symmetry. The mean field can be only a colour-neutral one, leaving us with the meson field as the only candidate for the mean field in baryons. Given that the interaction of constituent quarks with the chiral field is very strong, one can hope that the baryons' properties obtained in the mean field approximation will not be too far away from reality. It does not say that colour Coulomb or colour hyperfine interactions are altogether absent but that they

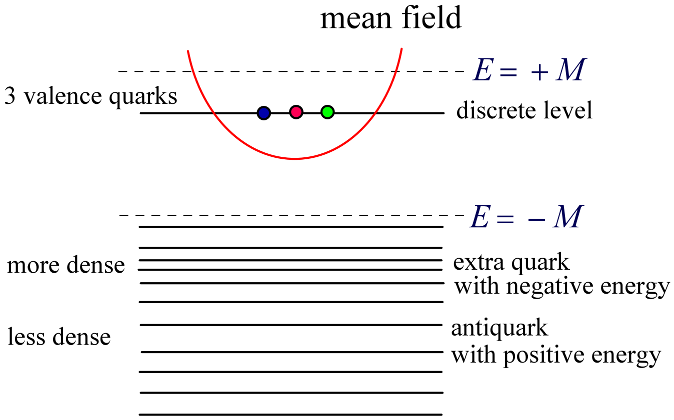

Fig. 3. A schematic view of baryons in the Mean Field Approximation. There are three 'valence' quarks at a discrete energy level created by the mean field, and the negative-energy Dirac continuum distorted by the mean field, as compared to the free one

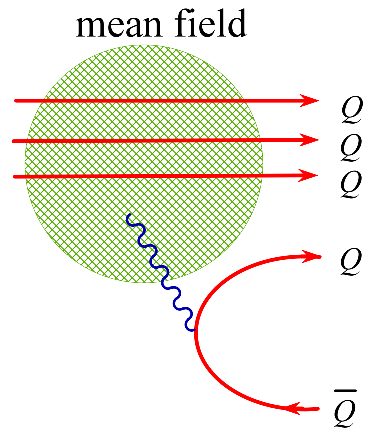

Fig. 4. Equivalent view of baryons in the same approximation, where the distorted Dirac sea is presented as quark-antiquark pairs. The number of $Q\bar{Q}$ pairs is proportional to the square of the mean field

can be treated as a perturbation, once the nucleon skeleton is built from the mean chiral field. Historically, this model of baryons [5] has been named the Chiral Quark Soliton Model, where the word "soliton" just stands for the self-consistent chiral field in the nucleon. Probably a more adequate title would be the Relativistic Mean Field Approximation to baryons. It should be stressed that this approximation supports full relativistic invariance and all symmetries following from QCD.

If the trial pion field in the nucleon is large enough (shown schematically by the solid curve in Fig. 3), there is a discrete bound-state level for three 'valence' quarks, $E_{\rm val}$. One has also to fill in the negative-energy Dirac sea of quarks (in the absence of the trial pion field it corresponds to the vacuum). The continuous spectrum of the negative-energy levels is shifted in the trial pion field, its aggregate energy, as compared to the free case, being $E_{\rm sea}$. The nucleon mass is the sum of the 'valence' and 'sea' energies, multiplied by three colours,

$$M_N = 3\left(E_{\rm val}[\pi(x)] + E_{\rm sea}[\pi(x)]\right). \qquad (1)$$

The self-consistent mean pion field binding quarks is the one minimizing the nucleon mass. If it happens to be weak, the valence-quark level is shallow and hence the three valence quarks are non-relativistic. In this limit the Mean Field Approximation reproduces the old non-relativistic $SU(6)$ wave functions of the octet and decuplet baryons, and there are few antiquarks [7]. If the self-consistent field happens to be large and broad, the bound-state level with valence quarks is so deep that it joins the Dirac sea. In this limit the Mean Field Approximation becomes very close to the Skyrme model which should be understood as the approximate non-linear equation for the self-consistent chiral field. Interesting, the famous Wess–Zumino–Witten term which is added "by hands" in the Skyrme model [8] appears automatically [5].

The truth is in between these two limiting cases. The self-consistent pion field in the nucleon turns out to be strong enough to produce a deep relativistic bound state for valence quarks and a sufficient number of antiquarks, so that the departure from the non-relativistic quarks is considerable. At the same time the mean field is spatially not broad enough to justify the use of the Skyrme model which is just a crude approximation to the reality, although shares with reality some qualitative features.

Being relativistic-invariant, this approach allows to compute all quark (and antiquark) distributions in the nucleon at low virtuality where they are not accessible in perturbative QCD. Important, all parton distributions are positive-definite and automatically satisfy all known sum rules [9]. This is because the account of the Dirac sea of quarks makes the basis states complete. The Relativistic Mean Field Approximation has no difficulties in explaining the "spin crisis" [10] and the huge experimental value of the so-called nucleon σ-term [11] – the two stumbling blocks of the naive quark models. Nucleon spin is carried mainly not by valence quarks but by the orbital moment between valence and sea quarks, and inside the sea. The σ-term is experimentally 4 times (!) bigger than it follows from valence quarks [4] because, again, the main contribution arises from the Dirac sea to which the σ-term is particularly sensitive. On the whole, the picture of the nucleon emerging from the simple Equation. (1) is coherent and so far has been adequate.

4 Nucleons under a microscope with increasing resolution

Inelastic scattering of electrons off nucleons is a microscope with which we look into its interior. The higher the momentum transfer Q, the better is the resolution of this microscope, see Fig. 5.

At $q < 300$ MeV one does not actually discern the internal structure; it is the domain of nuclear physics. At $300 < q < 1000$ MeV we see three constituent quarks inside the nucleon, but also additional quark-antiquark pairs; mathematically, they come out from the distortion

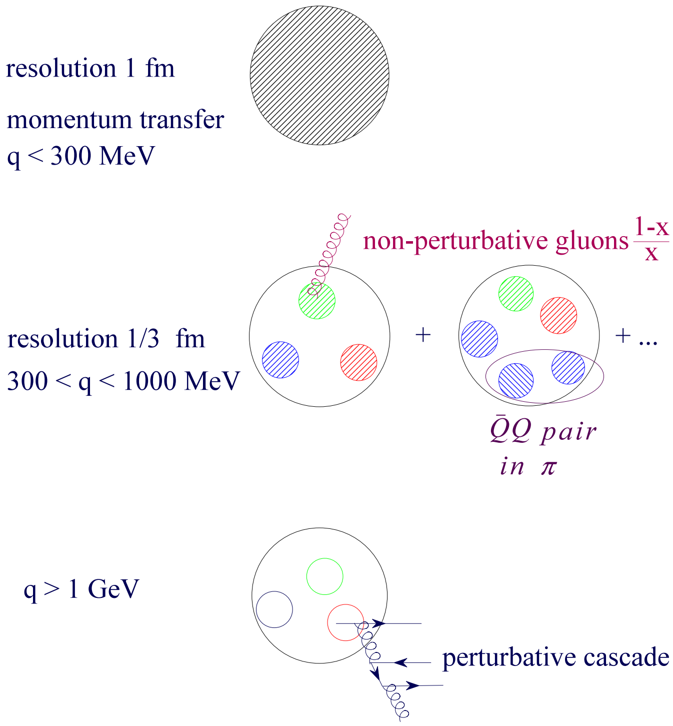

Fig. 5. Probing the nucleon with an increasing momentum transfer q

of the Dirac sea in Fig. 3. The appropriate quark and antiquark distributions have been found in [9]. In addition, the non-perturbative gluon distribution appears for the first time at this resolution. First and foremost, it is the glue in the interior of the constituent quarks that has been responsible for rendering them the mass, i.e. the glue from the instanton fluctuations. Interesting, these non-perturbative gluons are emitted not by the vector (chromoelectric) quark current but rather by the quarks' large chromomagnetic moment, and their distribution has been found by Maxim Polyakov and myself to be given by a universal function $(1-x)/x$, see Sect. 7 in [3].

At large $q > 1\,\text{GeV}$ one gets deep inside constituent quarks and starts to see normal perturbative gluons and more quark-antiquark pairs arising from bremsstrahlung. This part of the story is well-known: the perturbative evolution of the parton cascade gives rise to a small violation of the Bjorken scaling as one goes from moderate to very large momentum transfers q, but the basic shape of parton distributions serving as the initial condition for perturbative evolution, is determined at moderate q by the non-perturbative physics described above.

5 Pentaquarks

Based on this picture, Victor Petrov, Maxim Polyakov and I predicted in 1997 a relatively light and narrow antidecuplet of exotic baryons [12]; this prediction largely motivated the first experiments [13]. Both circumstances – lightness and narrowness – are puzzles for naive quark models.

Constituent quark models typically overestimate the mass of the exotic $uudd\bar{s}$ baryon (which I have suggested to name the Θ^+) by half-a-GeV, and it is clear why. One sums up five quark masses each about 350 MeV, adds 150 MeV for strangeness and gets something around 1900 MeV. In addition there is some penalty for the p-wave, assuming the Θ has positive parity. It gives more than 2 GeV. This is the starting point. Then one switches in his or her favorite interaction between quarks which may reduce the starting mass, but has to pay back the kinetic energy. Owing to the uncertainty principle, these two usually cancel each other to a great extent, even if the binding force is strong. Therefore, the Θ^+ mass of about 2 GeV is a natural and expected result in any constituent quark calculation.

The fundamental difference with our approach to pentaquarks is seen from Figs. 3 and 4. The fourth quark in the Θ^+ is just a higher density state in the Dirac sea: it has a *negative energy* $E = -\sqrt{M^2 + \mathbf{p}^2}$. One does not sum five quark masses but rather $(3M+M-M) = 3M$ to start with. This is because the extra $Q\bar{Q}$ pair in the pentaquark is added not in the form of, say, a vector meson where one indeed adds $2M$ but in the form of a pseudoscalar Goldstone meson, which costs nearly zero energy. The energy penalty for making a pentaquark is exactly zero in the chiral limit, had the baryon been infinitely large. Both assumptions are wrong but it gives the idea why one has to expect light pentaquarks. In reality, to make the Θ^+ from the nucleon, one has to create a quasi-Goldstone K-meson and to confine it inside the baryon of the size $\geq 1/M$. It costs roughly

$$m(\Theta) - m(N) \approx \sqrt{m_K^2 + p^2} \leq \sqrt{495^2 + 350^2} = 606\,\text{MeV}. \tag{2}$$

Therefore, one should expect the lightest exotic pentaquark around 1546 MeV. In fact one also adds an indefinite number of light pions to cook up the Θ^+. In the Dirac language of Sect. 2, the naive quark models attempt to make a pentaquark by adding a particle-hole excitation or a vector meson to the nucleon whereas in the world with the spontaneous chiral symmetry breaking there is a cheaper possibility: to add a collective excitation of the vacuum, i.e. the pseudoscalar meson(s).

Θ^+ is not a bound state of five good old constituent quarks: such bound states, if they exist, necessarily have a mass about 2 GeV. At the same time it is not a KN molecule – first, because its size is only about $\sqrt{2}$ larger than that of the nucleon [14], second, because it is an excitation of the pion field as well, third, because its coupling to the KN state is very weak. It is a new kind of a state. What is the giant resonance or a rotational state in a nucleus made of? Probably, it is the simplest to think of the Θ^+ as of a rotational excitation of the mean chiral field in the nucleon [12]. However, it also has a definite $5Q$-component wave function [7].

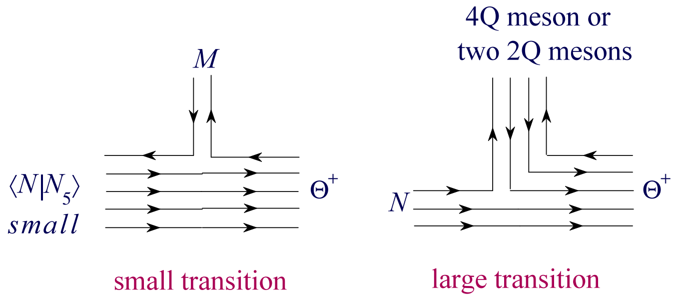

Fig. 6. All transitions between a $3Q$ and a $5Q$ baryon are suppressed if mediated by a $Q\bar{Q}$ mesons but may not be suppressed if it is a meson with a big $Q\bar{Q}Q\bar{Q}$ component

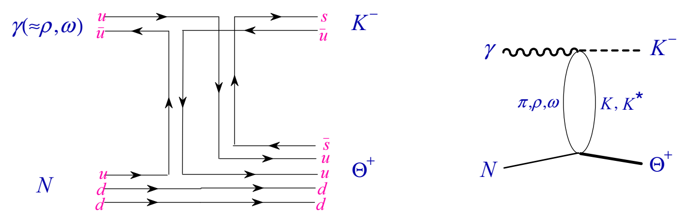

Fig. 7. Possible production mechanism of the Θ at low energies. There is no big suppression for a photon going into three mesons, for example

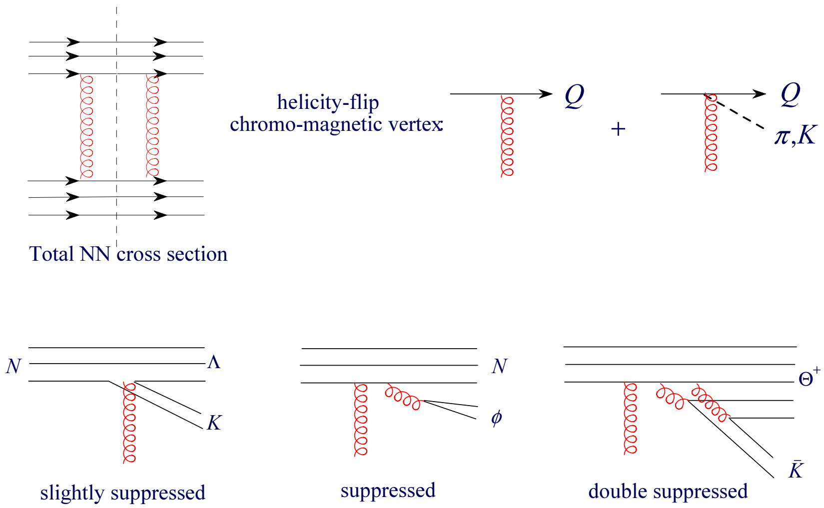

Fig. 8.

6 Production of pentaquarks

At present a dozen experiments have seen the Θ^+ and several, all at high energies, have not [1]. Apart from very different kinematical ranges, experimental cuts and techniques used in various experiments, which have to be carefully analyzed on the case-to-case basis, there might be some general physics behind the production (or non-production) of exotic baryons.

We know that Θ must have an extremely small width $\sim 1\,\mathrm{MeV}$ and hence a very small axial transition coupling [4]. By the same argument, its couplings to other $Q\bar{Q}$ currents are also unusually small, as is the magnetic transition moment [14]. The general reason for this suppression can be understood in the light cone quantization where only the transition to the $5Q$ component of the nucleon is allowed, which is suppressed by itself. However, the suppression needs not be so strong for the $\Theta^+ \to N$ transition via a non-local $Q\bar{Q}Q\bar{Q}$ current, for example in the form of the scalar $K\pi$ resonance $\kappa(800)$ or just of the continuum $K\pi$, $K\pi\pi$... states in the GeV region, see Fig. 6.

The Θ production mechanism at low energies could, then, look as shown in Fig. 7, where the exchange is either of a meson with a significant $4Q$ component (like $\kappa(800)$) or of more than one "normal" mesons.

At high energies, all single- and double-meson exchanges die out, and only the flavor-neutral gluon exchange survives. At low momenta transfer the gluon probably couples to the quark via a helicity-flip chromomagnetic vertex [3], see Fig. 8. In the same vertex a pion or a kaon can be emitted without considerable suppression, as it is a chirality-odd vertex. Therefore, ΛK production at high energies is only mildly suppressed. The production of the OZI-forbidden ϕ-mesons is suppressed by about an order of magnitude with respect to that of ΛK, as one has to create an extra $Q\bar{Q}$ pair. Θ requires a production of two $Q\bar{Q}$ pairs. According to Fig. 8, the production of the Θ could be, roughly, in the same proportion to ϕ as ϕ is to Λ. Therefore, the upper limit of the Θ to $\Lambda(1520)$ production ratio of 10^{-2} found by a careful analysis of the SPHINX data [15] may not be altogether unexpected. To my understanding, other high energy experiments report less stringent bounds. Similar conclusions have been reached in [16].

[1] See the Proceedings of the Workshop *Pentaquark-04*, SPring-8, Osaka, July 20-23 2004, to be published by World Scientific, for an extensive discussion.

Acknowledgements. I thank the organizers of the Elba Workshop on electron-nucleus scattering for hospitality. This work has been supported in part by the US Department of Energy under contract DE-AC05-84ER40150.

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